MODALISING PLURALS

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In some influential articles during the 1980s George Boolos proposed an interpretation of monadic second-order logic in terms of plural quantification [4] [3]. One objection to this proposal, pressed by Timothy Williamson [17,], focuses on the modal behaviour of plural variables, arguing that the proposed interpretation yields the wrong results in respect of the modal status of atomic predications. In the present paper I will present this objection and argue against it. In the course of developing the argument, I will have cause to consider the under-investigated question of how a logic for plurals should be extended to incorporate modal operators.

1. A Problem With Predication

To understand the modal objection to Boolos' interpretation of monadic second-order logic (MSOL), it is helpful to consider Rayo's formalisation of the key clauses from Boolos' translation scheme [11, 26]:

$$Tr'(X_j x_i) = x_i \prec xx_j$$
$$Tr'(\exists X_j.\phi) = \exists xx_j.Tr'(\phi) \vee Tr'(\phi*)$$

'where $\phi*$ is the result of substituting $x_i \neq x_i$ everywhere for $X_j x_i$'.

Here '*xx*' is a plural variable, taking as its values some things together in plurality, and '$\prec$' denotes plural inclusion. We read '$x \prec xx$' in inelegant logicians' English as '*x* is among *xx*'. Throughout this paper I will follow Rayo in taking the background (non-modal) plural logic[1] to be PFO$^+$, although for reasons which will become clear in due course I will follow McKay [9] in writing '$xx \approx yy$' to mean '*xx* are the same as *yy*', carefully distinguishing the *same things* relation from singular identity, denoted '='. There are good, although in my view not defeating, objections which can be made to the second of the Boolosian translation clauses. Our focus here, however, is on the first. This invites us to understand predication in terms of plural inclusion. But – the objection now announces itself – this gives us incorrect results when we consider the (alethic) modal behaviour of predications. Plurals conform to the modal principle $x \prec xx \rightarrow \Box x \prec xx$. The

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[1]For details see [11] and [8]. More general issues around the development of a formal logic of plurals are discussed in [9].

corresponding principle for predication – $Xx \rightarrow \Box Xx$ – is clearly false. So predication must involve something quite distinct from plural inclusion, not just at the superficial level of natural language grammar, but in terms of fundamental logical strucutre. Boolos' proposed translation scheme must be incorrect.

Here are two bad ways to respond to the modal complaint against Boolos. The first is to claim the objection has no force since Boolos is offering us a translation of MSOL, and the language of MSOL does not contain modal operators. Thus any objection to his translation on the grounds that it involves an unacceptable account of the behaviour of modal operators is entirely beside the point. This response has some value as a salutary reminder to philosophers that any serious study of formal logic should involve a rigorous specification of the language of the system under consideration That having been said, it remains the case that the proposed defence misses a vital point. One supposed virtue of the Boolos interpretation of MSOL is that it safeguards the system's status as part of logic, the thought being that plural inclusion and quantification do not bring with them ontological commitments injurious to logical status and do not require cognitive resources beyond those which might be thought logical[2]. It is a vexed question what precisely logicality consists in (if it consists in anything at all[14,]; however, a broad consensus could be assembled around the maxim that logic is a topic neutral enquiry. Logic can be used to talk about anything. But there are many topics where modal considerations loom large in our everyday language and thought. An obvious example here is the study of ordinary concreta: I am a philosopher, but I might have been a lion-tamer. My coffee cup is full at *t*, the present moment; alas, it might have been empty. It would be curious indeed if a supposedly topic neutral non-modal logic could not be extended in a straightforward fashion in order to facilitate formalisation of these sorts of claim.

Topic neutrality is one motive for rejecting a second bad response. This proceeds by noting that Boolos' motives for his plural interpretation were confined to the foundations of mathematics – he wished to allow for a non paradox-entailing version of ZF with only finitely-many axioms, for example. Modal considerations, the response continues, are simply irrelevant in mathematics; the truths of mathematics are necessary truths, and mathematical falsehoods are necessary falsehoods. Again, the criterion of topic neutrality blocks this response. MSOL might be especially useful for talking about mathematical entities, but that in no way means that it shouldn't be available for talking about other things as well. Indeed, if it is truly logic it *must* be so available; if it can't be used to talk about cabbages and kings, it

[2]The primary source of the felt need to defend the logicality of second-order logic is, of course, Quine[10], although see also [15]. For scepticism about the invocation of the Boolos interpretation as a defence of logicality see Resnik's [12] and Linnebo's [7].

isn't available *as a logic* to talk about sets and real numbers. In actual fact, the response faces trouble even with respect to its claim that mathematics is modally uninteresting. The modal structuralism [6] follows Putnam in claiming that ordinary mathematical statements are implicitly modal. More recently Linnebo has explored the possibilities of a modal foundation for set theory[].

Unsurprisingly, neither bad response is any good. We must do better if we are to defend the Boolosian interpretation of MSOL, and doing better will have to involve attacking head on the claim that the interpretation yields $Xx \to \Box Xx$. This necessitation of atomic predication is clearly unacceptable. Our only option then is to undermine the claim about the modal behaviour of plural variables which, in combination with Boolos' translation scheme, entails the unwelcome result about predication. Call the claim $x \prec xx \to \Box x \prec xx$ the *necessary inclusion thesis*, (NecInc) for short. What reasons are there to believe (NecInc)? It is certainly not a theorem of any system which would result from extending a standard plural logic by the addition of modal operators and any of the usual axiom systems for normal modal logics. What we do have as a theorem is:

(NecExt) $$\Box\big(\forall xx \forall yy\, xx \approx yy \leftrightarrow \forall x(x \prec xx \leftrightarrow x \prec yy)\big)$$

Which is the necessitation of the extensionality axiom for plural logic, and a **K**-thoerem. Now, (NecInc) is not entailed by (NecExt), although the latter *is* entailed by the stronger principle[3]:

(StrongNec) $$\Box\big(\forall xx \forall yy\, xx \approx yy \leftrightarrow \forall x \Box(x \prec xx \leftrightarrow x \prec yy)\big)$$

Something should be said at this point on the question of contingent existence. One immediately obvious ground for rejecting (NecInc) would be that one believed that some of the actual *xx*s exist contingently. Were this the case, (NecInc) will be false, even if one believes that the plural variable '*xx*' is a rigid designator. This avenue for rejecting (NecInc) does not get to the philosophical heart of the matter, however, which is precisely the question of the modal behaviour of plural variables[4]. This issue can be neutralised readily through Rumfitt's formulation of (NecInc)[5] [13, 113]:

(NeutrNec) $$x \prec xx \to \Box(E^2 xx \to x \prec xx)$$

[3]*Proof*: (1.) $x \prec xx$ (Ass. for $\to I$), (2.) $xx \approx xx$ (PFO), (3.) $\Box x \prec xx$ (2, (StrongNec)), (4.) $x \prec xx \to \Box x \prec xx$ (1-3, $\to I$). $\Box$.

[4]Any view on which plural variables designate rigidly is going to deliver 'wrong' results about predication given the plural interpretation of MSOL. Even making allowances for contingent existents, we will get the result that necessarily if all the things which are actually *X* exist, and *x* is actually *X*, then *x* is *X*. But this is clearly untrue. All the things which are actually red could exist without my nearest post-box (which is in fact red) being red.

[5]I have altered Rumfitt's notation to bring it into line with that used here.

Where 'E^2' is a plural existence predicate. The ready availibility of (NeutrNec) allows the question at issue to be framed in terms acceptable to all. For ease of presentation, my discussion here will be in terms of (NecInc), but nothing of present philosophical importance turns on this choice. As we have seen, (NecInc) is not a theorem of the bare system which results from modalising a plural logic. The introduction of (NecInc) to a system - either as an axiom itself, or through the addition of something like (StrongNec) as an axiom so as to ensure (NecInc)'s theoremhood - is a move which requires philosophical motivation in order to be principled. Our attention now turns to considering what such motivation might consist in.

2. Arguments for the Necessary Inclusion Thesis

It might appear as though there is a straightforward argument to be had for (NecInc). This begins by noting that, at least if we permit ourselves the logical resource of infinitary disjunction, an instance of (non-modal) plural inclusion will be equivalent to that of some disjunction:

(DIS) $$x \prec yy \leftrightarrow \bigvee_{\forall i} (x = y_i)$$

Where '*i*' ranges over some ordinals[6]. In order for (DIS) to hold for all the pluralities the Boolosian is interested in – crucially, for the sets (all of them) – the infinitary resources required will be substantial. The logic with the least expressive power suitable for the task is a plural version of $L_{\infty\omega}$, admitting arbitrarily large infinite disjunctions and conjunctions. Whilst this language in its non-plural form has been the object of logical study, and whilst extending it to admit plurals will be straightforward, there may be philosophical questions about the legitimacy of infinitary resources. We will not engage with these here.

The next move is to claim that (DIS) is definitional of inclusion; to be one of *these* simply is to be either *this*, or *this*, and so on. This is used to licence the necessitation of (DIS):

(NDIS) $$\Box\big(x \prec yy \leftrightarrow \bigvee_{\forall i} (x = y_i)\big)$$

Apart from (NDIS), the other premiss required is the more familiar assertion that identity is necessary[7]:

[6]Care is needed here. Considering (DIS), assign the empty set to '*x*' and the ordinals to '*yy*'. If we take '*i*' to range over a set, we incur the Burali-Forti paradox. My own view is that it is best to understand the range of '*i*' itself *plurally*. So, in the case under consideration, '*i*' ranges over the ordinals (all of them). More standard would be the invocation of proper classes.
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(NIDN) $x = y \rightarrow \Box x = y$

Given these premises, (NecInc) follows in **T**.

Proof.

(1) $x \prec yy$ (Ass. for $\rightarrow I$)

(2) $\bigvee_{\forall i}(x = y_i)$ $(1, \text{NDIS}, \leftrightarrow E, \rightarrow E, \text{T})$

(3) $\bigvee_{\forall i}\Box(x = y_i)$ $(2, \vee E, \text{NIDN}, \vee I)$

(4) $\Box x \prec yy \leftrightarrow \Box\bigvee_{\forall i}(x = y_i)$ (NDIS)

(5) $\Box x \prec yy$ $(4, \leftrightarrow E, 3, \rightarrow E)$

(6) $x \prec yy \rightarrow \Box x \prec yy$ $(1 - 5, \rightarrow I)$

□

The vulnerable point in this chain of argumentation is (NDIS).It is unclear why we should believe the claim it makes, unless we already believe (NecInc). But (NecInc) is precisely what we are seeking to establish through the invocation of (NDIS). The justification, mooted above, that (DIS) is *definitional* of inclusion and that its necessitation ought to be admitted, is question-begging. In itself (DIS) is platitudinous; there is nothing which is amongst some things without being a particular thing amongst those things. Conversely, if something is either y_1, or y_2, through to y_i, then there are some things which it is amongst. It does not follow that (DIS) is definitional of inclusion. For that to be the case would require that no thing could be amongst *these* without being *this*, or *this*, and so on. Yet this is what the supporter of (NecInc) needs to show. Perhaps some philosophical motivation for necessitation might be hoped for through the drawing of a connection between (singular) identity and *plural identity*.. The thought here will be that acceptance of (NecInc) opens the door to understanding plural identity as nothing over and above repeated instances of identity between objects. This position might draw some succour both from the belief that (StrongNec) may be viewed as a statement of modal plural identity conditions and from the fact that (StrongNec) follows from (NecInc) in a **B**-logic equipped with a plural existence predicate[8]

Superficially attractive though the stance which draws an intimate connection between singular and plural identity might be, it is mistaken. There is no such thing as plural identity, so in particular there is no such thing as plural identity which can be intimately connected to singular identity (or *identity* as we should call it). Integral to Boolos' motivation in developing

[8]Rumfitt derives $(E^2xx \wedge \neg x \prec xx) \rightarrow \Box\neg x \prec xx$ [13, 115]. From this plus (NecInc), already assumed for the derivation, (StrongNec) follows immediately.

the logic of plurals was their ontological neutrality. Our talk of the set of the three wise men is committed to the existence of a fourth entity, the set, over and above the magi. Our plural talk about them does not incur a similar commitment[9]. There are no such things as pluralities, however convenient the grammatically singular term 'plurality' might be for talking about some things together. It follows that plural variables are not eligible to flank an identity predicate, and so that '$\approx$' is not an identity predicate. As we have already remarked '$\approx$' denotes a sameness relation. It is the modal behaviour of that relation into which we are enquiring.
Now, the supporter of (NecInc) who sought to draw on a close relationship between $\approx$ and $=$ in support of her position might protest that, whilst the considerations mooted in the previous paragraph disagree with the letter of her position, they concur with its spirit. It is precisely because there are no such things as pluralities, she can be imagined as saying, that we can only understand '$\prec$' in terms of '$=$'. To say this, she will conclude, is to *define* '$\prec$' in terms of identity plus disjunction, and to to licence the necessitation of (DIS). Stalemate.

If progress is to be made, a new strategy is required. Presently I will survey some examples from natural language of rigid plural terms, which might be thought to count in favour of the rigidity of plural variables. I will then go on to draw attention to non-rigid plural terms in natural language, and will cite these in support of not viewing plural variables as rigid designators. Before that, we should pause and ask why evidence from natural language is useful for making progress in this area.

2.0.1. *How should we assess considerations from natural language?* We want to know whether '*xx*' is a *de jure* rigid designator[10] (DJRD). I am proposing examining natural language as a way of making progress in this debate. There is an objection which has some force against this strategy. The plural variable '*xx*' is a item from the lexicon of a formal, aritificial, language. The motivation for our present investigation is that we wish to use plural variables to interpret another formal, aritificial language - that of MSOL. But given that our concerns are exclusively with formal languages, and the correct formal semantics for these, why do we need to give any consideration to natural language? Can we not just stipulate that plural variables are not DJRDs, supply a model theory for modal plural logic which

[9]Like almost everything in the philosophical dispute over plurals, this has not gone uncontested. See [12]. I concur with (what I take to be) the majority position in taking Boolos' *Cheerios argument* to be decisive here [,].

[10]A *de jure* rigid designator is a linguistic item which designates rigidly in virtue of the semantic category to which it belongs. For example, a major thesis of Naming and Necessity is that singular proper names are *de jure* rigid designators. Some NPs are *de facto* rigid designators, in spite of not being *de jure* rigid designators. Consider 'the even prime'; this cannot denote anything other than the natural number 2, but yet the members of its semantic category (singular definite descriptions) are not in general rigid designators.

expresses this stipulation, and use the resulting system to interpret modal MSOL?

We can, of course, devise a system with variables 'xx_1', 'xx_2' etc. which do not designate rigidly, and proceed to use this system to interpret second-order logic. However, a major part of the philosophical appeal of Boolos' interpretation of MSOL is that it is widely (although not universally [7]) believed to have secured the *logicality* of the second-order system through interpreting it using, self-evidently logical, plural locutions. What, if anything, logicality consists in is a vexed question, but a frequent thought is that logic enjoys a cognitively basic status[11], and that the occurence of a candidate logical form in natural language has evidential value with respect to its being appropriately basic. Suppose now that we are considering a defence of the plural interpretation of MSOL which denies that plural variables are DJRDs, and so permits us to deny (NecInc). A conceivable worry is that, whilst non-rigid plural variables are readily entertainable model theoretically, they lack the cognitively basic status required for logicality. A response to this worry would be to point to the occurence of non-rigid plural terms in natural language. On the other hand, a way of weightening the worry would be to argue that there are no such terms.

What is meant by 'plural terms' in this context? Our ultimate interest is in plural variables in a formal language. The nearest natural language correlates to these, in the case of free variables, are plural demonstratives: 'these', 'those', 'ces', and so on, as my previous usage indicates. The function of demonstratives (and here the analogy with variables is apparent) is to stand in for terms of other sorts: compare 'The Western Isles are remote' with 'These are remote' and 'Those are remote'. Similarly, bound plural variables correspond to plural pronouns, which also have a place holding function: compare 'Les «Beatles» sont formidables' with 'Ils sont formidables'. The modal status of plural demonstratives and pronouns is, however, no clearer than that of their formal correlates – as our previous discussion of the rigidity issue in natural language terms indicates[12]. There is an important contrast here with singular demonstratives and pronouns, the status of which as rigid designators is uncontroversial. This lack of clarity need not represent an *impasse*, though. Given that we have noted the place-holding function of demonstratives and pronouns, we can make progress by addressing the modal status of admissible substituends for these categories, and by making an inference back from our conclusions about these to the modal status of plural demonstratives and pronouns themselves. Our

[11]See e.g. Linnebo's [7,]

[12]Although, we will see below a prima facie example of a non-rigid plural demonstrative phrase, which becomes compelling once certain philosophical arguments against non-rigid plural terms are answered.

attention turns, then, to those plural terms which are not themselves demonstratives or pronouns. With Rumfitt [13, 86], I will include amongst these compound names, such as ‘Amelie et Bernard’, collective names, such as ‘Radiohead’, and plural indexicals, such as ‘we’. I exclude plural definite descriptions from consideration[13], although as in the singular case (‘the Holy Roman Empire’) there are collective names which commence with the definite article. Here is Rumfitt on ‘the Channel Islands’:

> This expression does not mean ‘the islands in the English Channel’. The Isle of Wight is an island in the English Channel, but it is not one of the Channel Islands. Rather, the term refers to the islands in a certain archipelago off the western coast of Normandy. [13, 120]

We shall return to ‘the Channel Islands’ shortly.

2.0.2. *Natural Language Plural Rigidity.* It is immediately clear that one subcategory of plural term is *de jure* rigid, namely compound names. Nobody other than Alice or Bob could have been one of the things referred to by ‘Alice and Bob’, and each of Alice and Bob could not but have been one of the things referred to by ‘Alice and Bob’. Once we move beyond compound names, matters are far less transparent. Rumfitt argues in favour of the rigidity of collective names, and against some considerations which might be thought to support the claim that these are non-rigid. I will now outline the considerations which are Rumfitt’s target, and will then present, and cast doubt on, Rumfitt’s counter-argument.

Here is one reason one might suppose collective names not to be DJRDs. As we have already seen, ‘the Channel Islands’ is a collective name. Now here are two sentences which seem to express truths: ‘Herm might not have been one of the Channel Islands’ and ‘There might have been another one of the Channel Islands’. The first, on the face of it, is true because it is possible that Herm not be in its actual location but isolated in the Channel. If this were the case, Herm would exist but would not be one of the Channel Islands. The truth of the second appears immediate: is it not possible that there be an island in the archipelego additional to the ones there actually are. Where ‘aa’ is a collective name for the Channel Islands, we then have:

(HERM) $$h \prec aa \wedge \Diamond\neg(h \prec aa)$$

and,

(EXTRA) $$\exists x\, \neg(x \prec aa) \wedge \Diamond(h \prec aa)$$

(HERM) entails the negation of the universal closure of (NecInc). Both formulae involve ‘aa’ functioning non-rigidly.

[13] I take it that these are not terms. Even if readers demur on this point, they will presumably concede that, special cases aside (‘the even numbers’), plural definite descriptions are not rigid.

Rumfitt argues that the appearance of truth is deceptive in the case of both sentences. In the first case, he argues that its having approximately its actual spatial location is an essential property of a geographical entity, so that no island which is isolated in the Channel could be Herm. If there were such an Island, but no island in Herm's actual position, then Herm would not exists (and so the Channel Islands would not exist.) Intuitions regarding essential properties are frequently difficult to adjudicate, but I cannot agree with Rumfitt on this: suppose that by some miracle the mass of rock referred to by 'Herm' were transported instantaneously fifty miles West, along with all its inhabitants. Would they not very naturally, and correctly, describe the situation as one in which Herm had changed location? But then approximate spatial location is not an essential property of Herm's afer all. In any case, one can readily come up with similar sentences whose truth doesn't turn on the pecularities of the metaphysics of geographical objects. Here is a true sentence: 'Charlie Watts might not have been one of the Rolling Stones'.

Rumfitt's approach to 'There might have been another one of the Channel Islands' presents more of a challenge. He suggests that the intuition of its truth rests on treating 'the Channel Islands' as being descriptive in a fashion incompatible with the claim that it is a genuine plural term. The argument here is that in order for 'There might have been another one of the Channel Islands' to be true, 'the Channel Islands' has to function as a disguised definite description, along the lines of 'the islands which are an archipelego comprising of Jersey, Gurnsey...'. So, we are faced on Rumfitt's reckoning with a choice; either the sentence in question is not true, or else 'the Channel Islands' is not a plural term. Either way we do not have a counter-example to the thesis that plural terms are not *de jure* rigid.

There are (at least) two ways in which one might respond to Rumfitt here. The first consists in, what one might call, *ostrich nonrigidism* about NPs such as 'the Channel Islands'[14]. This position begins from the position that our philosophical theories about language owe a duty to the intuitions of competent language users. It takes on board the strong intuition that it is obviously true that there might have been another one of the Channel Islands, but thinks that there are good reasons to deny that 'the Channel Islands' is a disguised definite description of the type described by Rumfitt. After all, can't one be a perfectly competent user of the phrase without being in a position to know a priori, say, that the Channel Islands are an archipelego in the English Channel? Yet if it is part of the *meaning* of 'the Channel Islands' that its denotation (if any) is an archipelego in the English Channel, surely one ought to be able to know a priori that the Channel Islands are an archipelego in the English Channel simply by examining one's

[14]My inspiration for this coinage is Armstrong's term 'ostrich nominalism' [1].

usage. The ostrich nonrigidist stops here. 'The Channel Islands' is a non-rigid term. There might have been Channel Islands which are not amongst the actual Channel Islands. More than this, the ostrich nonrigidist will not say. She offers us no theory of how a non-rigid plural term refers. Ostrich nonrigidism is a fall-back position. I note it in order to emphasise that the philosopher who is convinced that there a plural terms which are not rigid need not admit defeat if she cannot offer an account of how such a term might succeed in referring. This having been said, it is not difficult to feel as though there is something unsatisfactory about ostrich nonrigidism. What is the other option?

What I will call the *conditional reference theory* (CRT) about plural names holds that plural names refer directly, such that their contribution to the truth-apt content of sentences in which they occur is simply the objects to which they refer[15]. In particular, plural names have no descriptive content. Thus far, CRT says about plural names what the direct reference theorist says about singular proper names. The direct reference theorist, however, holds that proper names denote rigidly, whereas CRT allows for non-rigidity in the plural case. Motivated by intuitions about the kind of case we have discussed, the CRT theorist argues as follows: consider an arbitrary singular proper name 'a'. Counterfactually, by virtue of rigidity, 'a' refers to a. We determine which thing in that context a is by means of a's identification conditions. These are not part of the content of 'a'. They are, rather, criteria for (as it were) locating a counterfactually. So far, so much orthodox direct reference theory. Now: consider an arbitrary plural name aa. In a counterfactual context 'aa' refers to aa. Disanalogously with the singular case, however, this does not imply rigidity since different things might be aa counterfactually. How do we determine which things conterfactually are aa? There are *aa-conditions* which enable us to determine which things are aa. No more than in the somewhat analogous singular case, are these conditions part of the semantic content of 'aa'. Some amongst the community of language users may be explicitly aware of the aa-conditions, indeed they may have introduced the conditions by stipulation. Yet neither is the reference of plural names mediated by its associated conditions, nor is it up to language-users, rather than the world, which things are (actually, or counterfactually) aa.[16]. CRT is a direct and semantically externalist theory

[15]Immediately, then, the problem of empty plural names rears its head. My position is that there are no such names. Every genuine plural name, in a non-modal context, has a referrent. This finds expression in the model theory for plural modal logic expounded below. Defence of this position will have to take place elsewhere.

[16]Consider the plural name of some mountains 'the Munros'. This is not a description. After all, I can know that Mary has climbed every one of the Munros without knowing that Mary has climbed every Scottish mountain which exceeds 3000ft. in height. 'The Munros' was introduced by stipulation. It is up to the world, however, not to language users that the Munros exceed 3000ft in height. Yet, by CRT, it is not possible that there be a Munro with a height of 2000ft.

of the reference of plural names.

CRT requires more investigation and development than is possible here; we have here a programme for future research, not a completed theory of plural reference. Nonetheless, I think that CRT is a credible theory which respects intuitions about the behaviour of plural names. Crucially, in the current context, it provides a theoretical basis for disagreement with Rumfitt. Armed with this theory, let us assess where things stand with (NecInc).

3. AGAINST THE NECESSARY INCLUSION THESIS

A single counter-example will serve to falsify (NecInc). If we have:

(7) $$\exists xx\, x \prec xx \wedge \Diamond\neg(x \prec xx)$$

Then we have ⌜¬(NecInc)⌝. (7) is entailed by:

(8) $$x \prec aa \wedge \Diamond\neg(x \prec aa)$$

If the argument about (HERM) above is correct, and given the legitimacy of reading across conclusions from natural language to an account of the correct rules for a modal plural logic, then (8) is secure. Further evidence against (NecInc) may be supplied by amassing examples of plural terms which are not rigid, since the claim that all plural terms are DJRDs – as this finds expression in (NDIS) – compels acceptance of (NInc). CRT places the intuition that there are such terms (for instance, 'The Rolling Stones') on a sure footing. This cumulative case against (NecInc), in my view, suffices to justify rejection of the principle and gives us philosophical permission to develop a semantics for the modal logic of plurals in which variables are not *de jure* rigid. As it happens, however, I think there are further considerations which should encourage us in this development. Two deserve attention before we before proceed:

(1) **Non-rigid plural demonstratives**. These are of particular interests for present purposes, given the close similarity between natural language demonstratives and variables in formal languages. An example, owing to Dorothy Edgington, is given by Rumfitt:

> In remonstrating at your indiscretion in relaying to a crowd some gossip about Smith, I might say, 'You shouldn't have said that. If Smith hadn't been delayed, he would have been one of those people. [13, 120-1]

Rumfitt's response, as in the second 'Channel Islands' case is to argue that the sentence in question is either false, or doesn't involve the demonstrative functioning rigidly. His reasoning here is that if Smith could have been one of those people, there must be an answer to the question which of those people he could have been. If we imagine otherwise, this is because 'we surreptitiously imagine a fresh use of the demonstrative, made in circumstances in which Smith has joined the throng'. Note that Rumfitt is relying here on the natural language equivalent of (NDIS), a principle we have seen reason to doubt. Once we abandon the belief that someone couldn't have been one of those people without being *a*, or *b*, or *c* etc., the natural reading of 'if Smith hadn't been delayed, he would have been one of those people' as involving a non-rigid plural demonstrative commends itself.

(2) **The nihilist recourse to purals**. The *mereological nihilist* believes that there are only simples, and that no composite objects exist. This belief is likely[17] to issue in the belief that there are no such things as coffee cups or computers, but that there might be quarks or leptons - or whichever particles (if any[18]) the physicists end up telling us are fundamental. Call this position *microphysical nihilism*. An immediate, and unfortunate, seeming consequence of microphysical nihilism is that we end up with an error theory about swathes of quotidien ordinary language. As I type this I am quite prepared to affirm 'There is a coffee cup on my desk, near my computer' – and I am sure you would be prepared to assert a sentence with the same propositional content if you had perceptual access to the present contents of my room. Yet, there are – on the hypothesis of microphysical nihilism – no computers, or desks, or (alas) coffee cups. One strategy for avoiding such an heroic error theory, supported by van Inwagen [16], involves understanding prima facie singular NPs as in fact plural noun-phrases[19]. Now suppose if (unlike van Inwagen) I am a thoroughgoing nihilist about not only inanimate objects, but also about organisms. Then, unless I affirm a very particular sort of substance dualism about mind[20], the plural response to the error theory is going to move me to claim that 'Quine' is a plural term, referring to some simples arranged Quine-wise. But 'Quine', thus understood, cannot be rigid. Quine might easily have lacked a given molecule, and so its constituent simples. Now, the metaphysical theories here are deeply controversial, but it seems peculiar to suggest that they can be ruled out a priori simply because of an account of plurals[21]

[17]But not inevitably: the nihilist could consistently believe that coffee cups and computers are themselves simples. She then faces the unenviable task of accounting for our strong intuition that, if such things exist at all, they have parts.

[18]The possibility of *gunk* rears its head here.

[19]For a formal fleshing-out of this view see Hossack's [].

[20]Namely a version in which I am an immaterial simple mind which is somehow associated with material simples, which are not constituents of me.

[21]Thank you to Samuel Lebens for drawing my attention to the use of plurals by mereological nihilists.

The development of a semantics for modal plural logic in which plural variables are not *de jure* rigid designators is well-motivated. The motivating considerations are not without more general philosophical implications: for example, our argument thus far involves the rejection of the claim, associated with Gareth Evans, that it is characteristic of referential parts of language that they designate rigidly. The questions arise: what, if anything, is essentially a feature of all and only referring parts of language, if rigid designation isn't? Is reference better understood as a family resemblance concept, rather than a tightly delimited concept under which proper names and NPs with close semantic similarities, and nothing else, fall? These issues will have to be addressed elsewhere. In terms of our immediate concerns, it remains to cash out our investigations in terms of a semantics for modal plural logic.

4. A Non-rigid Variables Semantics for a Modal Plural Logic

It is straightforward to supply a model theoretic semantics for a plural language in which plural variables are not rigid. We present here a semantics for the language of PFO+, including singular and plural constants and supplemented with modal operators. For ease of exposition, we work with a constant domain semantics; no particular issues related to plurals arise in modifying what follows for the varying domain case. For details of varying domain semantics see any competent text on first-order modal logic, for example [5,].

One clarificatory point should be made before we commence the exposition of the model theory. For the sake of convenience, we will be assigning (non-empty) *sets* to plural constants and variables. This model theoretic convenience should not be understood as carrying with it the claim that plurals are really a means of making disguised singular reference to sets. Instead, the formal semantics should be understood as modelling circumstances in which plurals denote the elements of the sets in question. It would be an interesting exercise to attempt the formulation of a formal semantics that avoided this kind of recourse to sets, but this is beyond our present scope[22].

Define a frame as usual: $\mathcal{F} =< S, R >$, where S is a non-empty set and R a relation on the elements of S. A model $\mathcal{M}$ on $\mathcal{F}$ is defined as $\mathcal{F} =< S, R, D, I >$. Here D is a non-empty set and I an *interpretation* of the modalised plural language. I makes assigments to the non-logical vocabulary as follows:

- To each *singular constant* some $d \in D$.

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- To each *n-adic predicate*, for each $s \in S$, some $d \subseteq D^n$.
- To each *plural constant*, a non-empty set of ordered pairs P such that $\forall < x, y >\in P\, x \in S \wedge y \subseteq D \wedge \forall z\, (< x, z >\in P \to z = y)$
- To each *plural predicate*, for each $s \in S$ some $p \subseteq \wp(D)$.

Note that plural constants are not, in general, rigid. Next we define a *valuation* in $\mathcal{M}$. A valuation v assigns to each free singular variable 'x' some $v(x) \in D$ and to each free plural variable 'xx' some $v(xx)$, where $v(xx)$ is a nonempty set of ordered pairs, the first co-ordinate of each of which is an element of S, the second co-ordinate a non-empty subset of D. It is a constraint on valuations that, for an aribtrary plural variable 'xx' and for any $s \in S$, $v(xx)$ has no more than one element with s as its first co-ordinate. .

We can now define truth in a model. We write $\mathcal{M}, s \Vdash_v \phi$ for 'ϕ is true at s in $\mathcal{M}$ on valuation v', where $s \in S$. In what follows we write 't_n' for a *singular term*. Singular constants and singular variables are singular terms. We define $v(t)$ as $I(t)$ when 't' is a singular constant:

(1) For an n-adic predicate 'F', $\mathcal{M}, s \Vdash_v Ft_1 \ldots t_n$ iff $< v(t_1 \ldots v(t_n) >\in I(F, s)$.
(2) For a plural constant 'aa', $\mathcal{M}, s \Vdash_v t \prec aa$ iff $.\exists < m, n >\in I(aa)\, (m = s \wedge t \in n)$
(3) For a plural variable 'xx', $\mathcal{M}, s \Vdash_v t \prec xx$ iff $\exists < m, n >\in v(xx)\, (m = s \wedge t \in n)$.
(4) For a plural predicate 'FF' and a plural constant 'aa', $\mathcal{M}, s \Vdash_v FFaa$ iff $\exists < m, n >\in I(aa)\, (m = s \wedge n \in I(FF, s)$.
(5) For a plural predicate 'FF' and a plural variable 'xx', $\mathcal{M}, s \Vdash_v FFaa$ iff $\exists < m, n >\in v(xx)\, (m = s \wedge n \in I(FF, s)$.

We specify recursive rules for compound wffs in the usual fashion:

(1) $\mathcal{M}, s \Vdash_v \neg\phi$ iff $\mathcal{M}, s \nVdash_v \phi$.
(2) $\mathcal{M}, s \Vdash_v (\phi \wedge \psi)$ iff $\mathcal{M}, s \Vdash_v \phi$ and $\mathcal{M}, s \Vdash_v \psi$.
(3) $\mathcal{M}, s \Vdash_v \Box\phi$ iff $\forall u \in S$ if sRu then $\mathcal{M}, u \Vdash_v \phi$.
(4) $\mathcal{M}, s \Vdash_v \forall x\, \phi$ iff for every valuation w, which differs from v at most with respect to the assignment to 'x', $\mathcal{M}, s \Vdash_w \phi$.
(5) $\mathcal{M}, s \Vdash_v \forall xx\, \phi$ iff for every valuation w, which differs from v at most with respect to the assignment to 'xx', $\mathcal{M}, s \Vdash_w \phi$

Other connectives and operators are understood as abbreviations. We say that some formula ϕ is *true* with respect to[23] s in $\mathcal{M}$ iff for every valuation v, $\mathcal{M}, s \Vdash_v \phi$.

We define validity in a model, which we write $\mathcal{M} \vDash \phi$:

(9) $\mathcal{M} \vDash \phi$ iff ϕ is true with respect to every$s \in S$ in $\mathcal{M}$

Validity simpliciter is defined relative to a frame or class of frames – $\vDash \phi$ iff ϕ is valid in every model based on the frame(s). If confusion is likely to arise, the turnstile can be subscripted to indicate which frames validity is understood relative to. An understanding of consequence arises natural from our definition of validity:

> $\Gamma \vDash \phi$ iff for every $s \in S$ in every model $\mathcal{M}$ based on the relevant frames, just in case if every element of Γ is true w.r.t. s then Γ is also true at s.

It is easy to show the **K**-invalidity of (NecInc):

Proof. Let $\mathcal{F} =< \{0,1\}, R >$. Now consider a model $\mathcal{M}$. Let $R = \{< 0,1 >, < 1,0 >\}$ and let $D = \{\pi, e\}$. Consider a valuation v including $v(x) = \pi$ and $v(xx) = \{< 0, \{\pi\} >, < 1, \{e\} >\}$. With respect to 0, '$x \prec xx$' is true, but '$\Box x \prec xx$' is false, since $0R1$ and $\mathcal{M}, 1 \nVdash_v x \prec xx$. Hence (NecInc) is false with respect to 0. It follows that (NecInc) is is invalid in $\mathcal{M}$, and thus that it is **K**-invalid. □

It remains to prove that this semantics validates the distinctive axioms of PFO+. This is straightforward, and we omit the details. The axioms are:
Comprehension: $\exists x\, \phi \to \exists xx \forall x\, (x \prec xx \leftrightarrow \phi)$
Nonemptiness: $\forall xx \exists y\, y \prec xx$
Extensionality: $\forall xx \forall yy\, xx \approx yy \leftrightarrow \forall x(x \prec xx \leftrightarrow x \prec yy)$

5. Conclusion

So then, we have provided philosophical motivation for the suggestion that plural variables are not *de jure* rigid designators, and have constructed a model theory for a plural modal logic which implements formally this suggestion. (NecInc) is not valid in the resulting logic. If we use this logic to interpret MSOL, the complaint that Boolos' reading of MSOL involves commitment to '$Xx \to \Box Xx$' is defeated. That is by no means all that needs to be done in order to sure up the Boolosian reading, but it is not

[23]Or 'at s'. I demur from this terminology in the text in order to disassociate myself from the dubious metaphysics which frequently arises from thinking of the elements of S as possible worlds. After all, there are no possible worlds, so in particular there are no possible worlds available as elements of sets. Or at least, so we are prone to say before considerable exposure to a certain type of metaphysics. For the algebraic purposes of model theory anything whatsoever - my coffee cup, Cheryl Cole or the Taaj Mahal - can be elements of S.

without significance.

Nor is it without significance elsewhere. In a recent paper, Timothy Williamson invokes (NecInc) when discussing Boolosian readings of second-order logic[18, 56ff.]. (NecInc) divests plural readings of second-order logic of any value as means of blocking Williamson's argument in that paper, an argument that any modal second-order logic ought to contain the Barcan formula and its converse, and so entail necessitism (the doctrine that everything which possibly exists necessarily exists). Our (NecInc) free plural modal logic might open up new avenues of response to Williamson.

But detailed response to necessitism was not my purpose here. There are some papers which discuss necessitism. This is not one of those (although it might have been).

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