

A Cardinal worry for Permissive Metaontology

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Abstract

Permissivist metaontology proposes answering customary existence questions in the affirmative. Many of the existence questions addressed by ontologists concern the existence of theoretical entities which admit precise formal specification. This causes trouble for the permissivist, since individually consistent formal theories can make pairwise inconsistent demands on the cardinality of the universe. We deploy a result of Gabriel Uzquiano's to show that this possibility is realised in the case of two prominent existence debates, and propose rejecting permissivism in favour of substantive ontology conducted on a cost-benefit basis.

Quine declared the ontological question to be 'what is there?'. The answer was even briefer 'everything', with the metaphysical hard graft consisting in filling in the details. [5] Since *On What there Is* the majority strand in analytic metaphysics has devoted its attentions to the Quinean details, to assaying the existents. Thousands upon thousands of words have been devoted to the question of whether properties exist, others to which (if any) mereological fusions are among the furniture of reality, still others to the supposed existence of mathematical objects – to mention just three of the most prominent existence questions considered by metaphysicians.. This research project finds itself called into question, however, by a growing constituency of permissivists. For the permissivist, the existence questions commonly asked by metaphysicians – or at least a significant number of them – lack depth

and admit of purely trivial answers.

1

One sophisticated and recent version of permissivism is owing to Jonathan Schaffer. He holds that, in all of our example cases, the answer to ‘Do F s exist?’ is ‘of course’! Reading ‘ F s’ as ‘numbers’, for instance, it suffices to demonstrate the existence of numbers to note that there is a number between two and three. [6, 357] For Schaffer, the moral of the story is that metaphysics shouldn’t focus on existence questions – which are easily resolved in the typical cases, in favour of the existence of the disputed entities – but should instead turn its attention to the structure of reality, and in particular to grounding relations. Other permissivists are more uniformly hostile to substantial metaphysics.¹

In general define permissivism thus:

(Perm:) For all F , assuming that F s can be described without contradiction, F s exist.

Restrictions and mitigations of **(Perm)** are plentiful, for example a somewhat permissivist metaontology might replace ‘all’ with ‘most’. Schaffer himself insists that canonical descriptions of candidate F not include grounding information. It is also important that ‘contradiction’ be understood in a

¹Hofweber supplies one instance, thinking Schaffer’s alternative project unacceptably esoteric. [2].

broad sense, encompassing not simply sentences of the form $\lceil P \wedge \neg P \rceil$ but also, for instance, incompatible predicates. Permissivism need not commit a proponent to square circles.²

It might seem that **(Perm)** is obviously false. Let *phlogi* be units of phlogiston, such that any combustible body contains at least one phlogium. By **(Perm)**, it follows that phlogi exist. Yet, surely it is a settled result of chemistry that phlogiston does not exist. Therefore, the argument goes, **(Perm)** is false. But this is too hasty; the permissivist will retort, with an air of plausibility, that *of course* phlogiston exists, it is a theoretical posit. We quantify over it when engaged in scientific theory choice, and it is referred to by noun-phrases in true declarative sentences, such as ‘Phlogiston is a theoretical posit’. Unless we want to engage in the costly enterprise of rejecting classical quantificational logic, the permissivist case goes, we are committed to the existence of phlogiston. What we are not committed to is the existence of phlogiston *qua* concrete physical reality. *Prima facie* the response is a strong one.

That said, having laid permissivism on the metaontological table, I now want to argue that there is simple logico-mathematical reason that no version of **(Perm)** which permits useful progress in metaontology can be true.

²Of course a permissivist *could* be either a dialethist or a Meinongian, it’s just that we don’t want to build these positions into the *definition* of permissivism.

2

Permissivism, if it is to be of any value in directing metaphysical research, must surely adjudicate the key current ontological debates. An interesting feature of these is that they often, perhaps even typically, concern *theoretical entities*. By ‘theoretical entities’ I mean proposed entities which (a.) in key cases at least, are not objects of everyday experience and thought, and (b.) admit of precise specification in a formal theory. The first of these conditions is unsurprising. It is a far bolder philosopher who questions the existence of tables than of esoteric mathematical objects, since the existence of tables looks *obvious*³. Subtleties arise around cases where some proposed type of entities has tokens amongst the items of everyday encounter, but where these are atypical of the type. My table is one of the many fusions that a believer in unrestricted composition will admit to her ontology, but is atypical in being of any particular concern to human beings and (*modulo* concerns about determinacy) readily isolated as an object of reference. The question is whether there are, in general, unrestricted fusions or, alternatively, whether there aren’t, in spite of which my table (wrongly identified by the universalist as one of her fusions) exists. It is precisely because many existence debates concern objects whose existence is tracked by some kind of generation principle, like unrestricted composition, that clear formal specification is important for grasping what is at issue. Hence (b.).

Here are two existence debates of the sort I have in mind:

Mathematical realism: Mathematical platonists believe that

³For notorious dissent, see [11].

sets exist. Mathematical nominalists deny that sets exist.

Composition: Universalists hold that for any things, xx , there is a fusion of all and only the xx . Non-universalists deny this.⁴

Both are current debates in metaphysics, on which prominent philosophers adopt positions. Both enjoy the advantage of admitting discussion in terms of clearly formulated and well-understood mathematical theories, standard set theory and extensional mereology respectively. Permissivism holds out the prospect of resolving both debates quickly: *of course* there are sets, *of course* there are fusions. In both cases, after all, we can give a clear account of what is required for the postulated entities to exist – the existence of the elements, in the case of a set; that of the parts, in the case of fusions.⁵ What more could be required? The debate over mathematical realism should be resolved quickly in favour of the platonist, and that over composition in favour of the universalist. Rather than wasting any more ink on these debates, the metaphysician should turn her attention to other questions. Are parts dependent on the wholes they constitute, or *vice versa*? Should set membership be understood as the converse of a grounding relation?

Alas, there is trouble in permissivist paradise. As Gabriel Uzquiano has shown, in the context of another discussion, acceptance of a very natural theory of sets and a very natural theory of universal fusion, leads to contradiction if quantification is understood as absolutely general. [10] For suppose

⁴' xx ' here is a plural variable, ranging over some things in plurality. See [4].

⁵From this gloss, which is in the spirit of [3], it follows immediately that *nothing* is required of the world for the empty set to exist. A permissivist approach to the necessary existence of pure mathematical entities might be forthcoming.

that second-order ZFCSU⁶ is true. Suppose furthermore that the set membership relation is absolutely general, that anything can be an element. Then the cardinality of the universe is strongly inaccessible, that is to say it is of cardinality $\kappa > \aleph_0$, such that there is no $\lambda < \kappa$ where $\kappa = 2^\lambda$, and further that κ cannot be reached by taking unions of sets of smaller cardinalities. Now suppose that classical atomistic extensional mereology, formulated with plural (or otherwise higher-order) quantifiers, is true. Suppose additionally, that the part- whole relationship is absolutely general, that anything can be a part. Then the universe has cardinality 2^κ where κ is the number of atoms. Cantor's theorem secures that $\kappa \neq 2^\kappa$, and so specifically that the cardinality of the universe is not strongly inaccessible. Thus affirmation of ZFCSU and atomistic extensional mereology against a background of higher-order and absolutely general quantification leads straightforwardly to contradiction. It seems that we can't, with no further explanation, affirm the existence of *both* sets and fusions. And yet these are paradigm cases of the objects of ontological dispute; if permissivism can't help us here, it is of limited value indeed.

The permissivist faces difficulty, then, but it might be thought that the difficulty is not insurmountable. It is open to her to reject one or both of the logical preconditions for deriving the contradiction, higher-order logic or absolutely general quantification. In both cases, though, there is an unnatural feel to the abandonment. Higher-order quantification is well understood and essential for capturing key mathematical structures [7]; in particular, there would be a real loss were second-order quantifiers not admitted for the for-

⁶That is ZFC with *urelemente* and an axiom stating that the *urelemente* form a set.

mulation of set theory – as Boolos puts the point, the axioms of separation and replacement are ‘crying out’ for a second-order statement. [1, 65]⁷ On the other hand, the abandonment of absolutely general quantification in the cause of preserving a metaontological project appears self-defeating, since the very claims that project aspires to adjudicate are absolutely general in intended application. The universalist doctrine that *everything* is a part of some whole is not supposed to be about some restricted section of reality.

More promising is the suggestion that either the set theory or the mereology be modified in order to block the contradiction. The most obvious target is the axiom stating that the *urelemente* form a set, which is not standard fare in mathematical practice. This non-adoption of the *urelemente* set axiom, however, stems more from the question whether the non-sets form a set simply being one with which mathematicians are not greatly concerned, rather than from a considered rejection of the axiom. Once it *is* brought under consideration, it looks quite reasonable (surely there can’t be *that* many non-sets). In any case, mere rejection of the axiom does not resolve the difficulty over cardinality on the assumption that the pure sets are in 1-1 correspondence with the universe [10, 311]⁸. Perhaps, then, the mereology is the appropriate target for modification. Here the weak point is the insistence on atomicity - that there are no objects, all of whose parts have further

⁷An more moderate moderation of the background logic would involve adopting second-order logic with Henkin semantics, and then appealing to the downward Löwenheim-Skolem theorem to the effect that both the set theory and the mereology have countable models. The problem here is finding a non-textitad hoc motivation for abandoning standard semantics. Williamson supplies a recent defence of this semantics [12, 229-30].

⁸For an argument in favour of this assumption, see [9].

proper parts. Could there not, after all be *gunk*? [8] Again, abandonment of atomicity alone will not suffice to avoid the contradiction. This requires that the atomless sums be no fewer than strongly inaccessible in number [10, 315]. That there is *this much* gunk is a serious ontological claim, and not the kind of auxiliary premise we would expect to see imported at the stage of deciding on metaphysical *method*.

This is the key point here: it is not that there is no arguments to be had about the *urelemente* set, or about whether the universe is incredibly gunky, or about some other proposed modification of set theory or mereology in order to avoid the contradiction identified by Uzquiano. Instead, the problem is that once we engage in these arguments we are involved in substantial discussion of what there is, or what there might be. Recall from our earlier examination of debates about theoretical entities that formal specification is central to determining what is at issue in existence debates. To put the matter more precisely: for a theory Γ , the question whether the existence claims contained in the closure of Γ under logical consequence are true is a debate about the existence of a type of theoretical entity. When we discuss, for instance, whether sets which do not include an *urelemente* set exist, we are doing *ontology*. Whereas when we embarked on discussion of permissivism, we were supposed to be talking about *metaontology*. This in itself might not be a problem – maybe a holism which does not set up a clear boundary between metaphysics and the discussion of metaphysical methodology, but rather allows considerations from one to bear on the other, is the correct approach in this area.

Even so, the permissivist is now in an embarrassing position. For the kinds of questions raised in the attempt to rescue permissivism about sets and fusions from contradiction are precisely the kinds of questions from which permissivism was supposed to rescue metaphysics. Whether there are sets and fusions, and if so *which* sets and fusions there are, and which principles of composition can be appealed to in answering this latter question – these are familiar fixtures on the philosophical agenda. If a permissivist metaontology moves us to revisit them as a priority, it is clearly a lot less ground-changing than its proponents suggest.

3

Permissivism promises a generous ontology and the saving of philosophical labour. For a large class of candidate F , permissivists propose to answer the question ‘Do F s exist?’ with a swift ‘yes’ in accordance with **(Perm)**. The problem here, as we have seen is that commitments which might be individually acceptable – to sets with a general membership relation, or to fusions with a general parthood relation⁹ – and which the permissivist, in her own terms, should be expected to accept, ensnare us in contradiction when accepted jointly.

That commitments which are individually unproblematic may be contradictory in combination is unsurprising. It is, for example, the basis of the Bad

⁹There are other examples that could be given. Uzquiano himself instances Fine’s General Theory of Abstraction as making cardinality demands.

Company problem for neo-Fregeanism. Yet recognition of the possibility of mutually incompatible commitments is extremely damaging to permissivism. A principle such as **(Perm)** suggests a method of metaphysical enquiry where postulated existents appear *individually* before the ontological dock, each to receive admission into the catalogue of the universe. Against this, the moral of our contradiction is surely that commitments should be considered in combination. Metaphysics, like any science, needs to proceed by considering the way its various commitments interact, modifying or rejecting them in order to avoid contradiction, and making decisions about how to do this by weighing up costs and benefits. An immediate corollary is that the ontological question is not trivial. Assaying the existents is a substantial metaphysical task.

Keywords: metaontology, permissivism, set theory, mereology

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